Highlights of Spanish Astrophysics XII, Proceedings of the XVI Scientific Meeting of the Spanish Astronomical Society held on July 15 - 19, 2024, in Granada, Spain. M. Manteiga, F. González Galindo, A. Labiano Ortega, M. Martínez González, N. Rea, M. Romero Gómez, A. Ulla Miguel, G. Yepes, C. Rodríguez López, A. Gómez García and C. Dafonte (eds.), 2025

Music and astronomy. VI. Harmonia Mundi 2024

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Abstract

In 2009, Mike Oldfied published his *Music of the Spheres*, Depeche Mode their *Sounds of the Universe*, and Antonio Arias his *Multiverso*, being the last track of his album *Harmonia Mundi 2009*. In 1975, Laurie Spiegel composed the version of the *Music of the Spheres* that started the famous Voyager Golden Record. Almost five decades after Spiegel's sonification and more than four centuries after original Johannes Kepler's *Harmonices Mundi*, an astrophysicist and two celebrated indie rock musicians "de Graná" are revisiting the fifth chapter of Kepler's book. Previous versions had focused on the frequency of the planets; however, Kepler's hypothesis was that the sound of each sphere depends on a music interval that is a simple function of the planet's orbit eccentricity. Without leaving Granada, we relate how we are composing this new Harmonices Mundi, Music of the Spheres, Harmony of the Worlds, Musica Universalis, Harmonia tou Kosmou in 2024.

1 Introduction

Como hogueras en la noche
una por segundo la última en ejecutar
su danza de combustión en la Galaxia
provocó el éxtasis de Johannes Kepler
por siempre geómetra [...]

Desde una estrella enana, Natalia Carbajosa/Antonio Arias, *Multiverso* (2009)

Pythagoras defined three kinds of music: instrumental, human and celestial. In the words of David Byrne, alma mater of Talking Heads: "Celestial music, which we try to imitate – from which divine harmonies emanate – really exists and has its origin in the spheres that support the planets. Pythagoras believed that the planets were attached to rotating crystal spheres and that each planet, together with its crystal sphere, produced its own unique note as it whistled through the cosmic ether. Hence the Harmony of the Spheres... This great cosmic chord was so perfect that ordinary people could not hear it, except for a select few: Adam and Eve, Moses, Pythagoras and his acousmatic followers, the Zoroastrians... According to Saint Augustine, all men will hear that sound just before they die, at which time the secret of the cosmos will be revealed to them...".

In recent decades there have been several attempts to ring the planets' crystal spheres again. Without going any further, the last track on Antonio Arias' album *Multiverso* was titled *Harmonia Mundi 2009* and came close to revealing the secret of the cosmos. Not so close were Mike Oldfeld and his *Music of the Spheres* and Depeche Mode and their *Sounds of the Universe* (all three albums published in 2009!). But perhaps the closest attempt to reveal such a secret to us was that of Laurie Spiegel with her 1975 *Harmonices Mundi*, which opened the Voyager Golden Record.

However, all of these latest approaches to the Musica Universalis have been incorrect. Below we present a new solfeggistic-mathematical-astrophysical approach that extrapolates with current data what was proposed centuries ago by Platonic-Aristotelian, medieval and Renaissance philosophers, but developed in depth by the German mathematician, astronomer and astrologer Johannes Kepler in his book *Harmonices Mundi* (*The Harmony of the World*, 1619) [1].

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In the fifth and final chapter of *Harmonices Mundi*, Kepler enunciated his third law of planetary motion (that the cube of the semimajor axis is proportional to the product of the square of the orbital period times the sum of the masses). Kepler stated in that same chapter that "the difference between the maximum and minimum angular velocities of a planet in its orbit approximates a harmonic ratio". The most common example of this theory, and explained in depth in *Harmonices Mundi*, is the quotient between the speeds of the Earth at perihelion (maximum approach to the Sun, maximum orbital speed, at the beginning of January) and aphelion (maximum distance from the Sun, minimum orbital speed, early July). That ratio was 16:15 with the data that Kepler had at his time. It turns out that this variation is a Ptolemaic diatonic semitone, which was the musical interval between the notes E (mi) and F (fa) in just intonation. That's why the Earth sounds mi, fa, mi according to Kepler ("[...] you may infer even from the syllables that in this our home **mi**sery and **fa**mine hold sway"). The music of Mars and Jupiter plays at an interval 19:18. And so for the rest of the planets in the Solar System known then... The frequency assigned to a planet does not depend on its orbital period, but on another parameter of its orbit. To understand it, we need to first clarify certain astronomical and musical concepts:

- The Solar System has eight planets. At the time of Kepler, Herschel had not yet discovered Uranus, nor had Galle/d'Arrest/Le Verrier yet discovered Neptune. Ceres and Pluto, both considered planets at some point in history, are just dwarf planets, as are Eris, Haumea, and Makemake.

- The orbital speed of a planet v depends on the mass of the Sun M, the semimajor axis a, and the physical separation at a certain time r through the equation $v^2(r) = GM(2/r–1/a)$, where G is the universal gravitational constant.

- By simple geometry and definition of variables, the physical separation between the Sun and a planet is $r_{\rm per} = (1–e)a$ at perihelion, and $r_{\rm ap} = (1 + e)a$ at aphelion, where e is the orbital eccentricity.

- Substituting and developing, the quotient between velocities at perihelion and aphelion results in a very simple value that only depends on the eccentricity: $v_{\rm per}/v_{\rm ap} = (1 + e)/(1–e)$. To get an idea, with the values of the eccentricities known today, this ratio varies approximately between 1.014 for Venus (the least eccentric, i.e. with the most circular orbit) and 1.518 for Mercury (the most eccentric, i.e. with the most elliptical orbit).

- Each musical note corresponds to a frequency. Thus, A4 (la$_3$) corresponds to 440 Hz, which is considered the tuning pattern since the mid-20th century. C4 (do$_3$), the first "C" that we studied in school (middle C), corresponds to approximately 261.626 Hz. C5 (do$_4$), the last do when we sing "do-re-mi-fa-sol-la-si-do" (tenor C), corresponds to about 523.251 Hz, which is just double the frequency of C4. So, the musical interval between two sounds whose frequencies have a 2:1 ratio is an octave. In other words, an octave higher than a sound is one that has twice the frequency and a lower one is one that has half the frequency.

- In the 12 equal temperament system, a complete octave contains six tones or twelve semitones, which is the smallest of the intervals between consecutive notes in a diatonic scale (e.g., between E and F). A complete octave also contains exactly 1200 cents. A semitone on the tempered scale contains, therefore, 100 cents. The cent is the unit to quantify intervals and compare them in different tuning systems. Between two sounds separated by 1 cent (identical for human sound), their frequencies have an approximate ratio 1+1/1731:1. To calculate exactly how many cents n there are between two frequencies ν_1 and ν_2, the formulas $n = 1200 \log_2 (\nu_1/\nu_2) \approx 3986 \log_{10} (\nu_1/\nu_2)$ are used.

- Between the octave and the cent there are a multitude of musical intervals defined under different tuning systems. In Kepler's time, the most used systems were the Pythagorean tuning and the just intonation, while today the 12 equal temperament is mainly used. In the end, everything is physics, and the current intervals are the natural evolution of the ratio of the number of standing waves that Pythagoras and Euclid generated in their monochord, which was an ancient instrument with a soundboard and a single string.

In Table 1, we summarise and update Kepler's values with the most current data on the orbital eccentricity of the eight planets (he only did it with six), quotient of velocities in perihelion and aphelion, number of cents that would correspond to this quotient by equating orbital velocities with frequencies ($n_{\rm orb}$), and the frequency ratio and number of cents of the closest single musical interval in 12 equal temperament ($n_{\rm mus}$). The names of the intervals

Table 1: Orbital eccentricities, musical intervals and planet masses

Planet	e (J2000)	$\frac{1+e}{1-e} = \frac{v_{\rm per}}{v_{ap}}$	$n_{\rm orb}$ (cent)	Musical interval	$n_{\rm mus}$ (cent)	M ($M_\oplus$)	P (yr)
Mercury	0.205 630 69	1.5177	722.29	3:2	701.96	0.0055	0.240 846
Venus	0.006 773 32	1.0136	23.45	3^{12}:2^{19}	23.46	0.815	0.615 198
Earth	0.016 710 22	1.0340	57.86	25:24	70.67	1.0000	1.000 017
Mars	0.093 412 33	1.2061	324.38	6:5	315.64	0.107	1.880 85
Jupiter	0.048 392 66	1.1017	167.69	11:10	165.00	317.8	11.862
Saturn	0.054 150 60	1.1145	187.68	10:9	182.40	95.159	29.4475
Uranus	0.047 167 71	1.0990	163.44	11:10	165.00	14.536	84.0205
Neptune	0.008 585 87	1.0173	29.73	64:63	27.26	17.147	164.8

are given in the second column of Table 2. In reality, there are other intervals that are even more similar, but more complicated; e.g., for Mercury, which has the most eccentric orbit in the Solar System ($e \approx 0.206$), a "narrow diminished sixth" of 1024:675 ($n_{\rm mus}$ = 721.51 cent) fits better than a perfect fifth. Likewise, a "septimal third tone" of 28:27 ($n_{\rm mus}$ = 62.96 cent) also fits better than a just chromatic semitone for Earth. Besides, Kepler used a unison interval of 1:1 for Venus, while a Pythagorean comma (one of the smallest intervals, barely perceptible to the human ear) seems to sound better. As a comparison, the smallest "playable" interval on a perfectly tuned piano is a semitone ($n_{\rm mus}$ = 100 cent), unless one uses microtonal arrangements (as previously done by, e.g., Karlheinz Sockhausen's *Gesang der Jünglinge*, Wendy Carlos's *Beauty In the Beast*, or Radiohead's *Kid A*).

Going one step further than Kepler was, in the third column of Table 2 we assign each planet in the Solar System a height or, better, a voice type, which is feminine for the telluric or terrestrial planets, and masculine for the Jovian or gaseous planets. The more massive the planet, the lower the first note of the interval. The counterpart to the voice on a piano are the octaves.

In reality, each planet also has its orbital period: the shortest for the closest one (Mercury, 88 days, 0.24 years), the longest for the furthest one (Neptune, 164.8 years). The orbital, or sidereal, period of the Earth is one astronomical year, which is roughly 365.25 days. Therefore, each interval could also be assigned a duration approximately proportional to the planet's orbital period. If the Earth had the duration of one whole note, Mercury would have approximately one quarter note (a quarter of the Earth) and Neptune would instead have approximately 164 whole notes, one half note and one quarter note. Since performing several complete revolutions of Neptune would take a long time (although not as long as John Cage's *ORGAN*^{2}*/ASLSP (As Slow As Possible)*, a piece that takes 639 years to be fully interpreted), artistic license can be granted in the interpretation, such as assigning each planet the classical durations. Forgetting about the hundred twenty-eighth and two hundred fifty-sixth notes, we just assign: Mercury = sixty-fourth note, Venus = thirty-second note, Earth = sixteenth note, Mars = eighth note, Jupiter = half note, Saturn = whole note, Uranus = double whole

Table 2: Musical intervals, notes and voices

Planet	Musical interval	Voice	Note
Mercury	Perfect fifth	Soprano	Hemidemisemiquaver
Venus	Pythagorean comma (Unison)	Contralto	Demisemiquaver
Earth	Chromatic semitone	Contralto	Semiquaver
Mars	Minor third	Mezzo-soprano	Quaver
Jupiter	Major tone	Bajo	Minim
Saturn	Minor tone	Baritone	Semibreve
Uranus	Major tone	Tenor	Breve
Neptune	Septimal comma (Unison)	Tenor	Longa

note, Neptune = longa. The quarter note (crotchet) does not sound because, according to the Titius-Bode law, which is also a power law as in the definition of the intervals, it corresponds to the Main Asteroid Belt. We list the British name of note value corresponding to each planet in the fourth column of Table 2.

At the end of interpreting this Musica Universalis, according to Saint Augustine, the secret of the Cosmos will be revealed to the listener. And they will die.

3 Harmonia Mundi 2024

The previous sections in English are based on part of the content of the paper "*Música y astronomía. Púlsares, viajeras, musas, planetas, espaciotiempo y la armonía de las esferas*", which was published in Spanish [2]. In turn, this paper was partly based on the script of a *Longitud de onda* radio programme for Radio Clásica/Radio Nacional de España, broadcasted in March 2017, in which a version of the Harmonia Mundi was performed live by Daniel Quirós on a grand piano [3]. The 90 min programme can be heard in this podcast. That version of the Harmonia Mundi was performed again in May 2017 at the Museo Nacional del Teatro in Almagro, Spain, with Alfonso Candelas on the grand piano. As far as we know, no listener died because of these performances. To close the circle (or, rather, the circle of fifths in music theory), the script of the radio programme was based on a short entry for a Spanish outreach journal [4].

Taking advantage of the meeting of the Sociedad Española de Astronomía being held at authors J.J.M. and A.A.'s home city and of the previous collaborations between them and J.A.C. on communicating astronomy through rock music [5, 6], the three of them planned a revision of Harmonia Mundi 2017 with a 19th-century grand piano, 21st-century recording technology and perhaps some exotic instrument (for the solar wind, micropulsations, supergranulation and spots). However, at the moment of writing these lines, the authors are touring through Spain for celebrating the 35th anniversary of Lagartija Nick, so the revision of Harmonia Mundi may be delayed to 2025, when the secret of the cosmos will be revealed.

Acknowledgments

This contribution was presented at the symposium *Exploring New Horizons: Transforming Astronomy Through Sonification and Multidisciplinary Innovation*, organized by Rubén García-Benito (chair, IAA-CSIC), Sandra Pauletto (co-chair, KTH), Enrique Pérez-Montero (IAA-CSIC) and Anita Zanella (INAF). We acknowledge financial support from the Spanish Agencia Estatal de Investigación (AEI/10.13039/501100011033) of the Ministerio de Ciencia e Innovación and the ERDF "A way of making Europe" through project PID2022-137241NB-C42.

References

[1] Kepler, J. 1619, *Ioannis Keppleri harmonices mundi libri V : quorum primus geometricus ... secundus architectonicus ... tertius proprie harmonicus ... quartus metaphysicus, psychologicus et astrologicus ... quintus astronomicus & metaphysicus ... : appendix habet comparationem huius operis cum harmonices Cl. Ptolemaei libro III cumque Roberti de Fluctibus ... speculationibus harmonicis, operi de macrocosmo & microcosmo insertis*

[2] Caballero, J. A. 2020, Torre de Los Lujanes, 75, 91

[3] Caballero, J. A., Blázquez, F., Criado, Y. 2023, Highlights of Spanish Astrophysics XI, Proceedings of the XV Scientific Meeting of the Spanish Astronomical Society, 512

[4] Caballero, J. A. 2014, Astronomía, 183, 70

[5] Caballero, J. A., Arias, A., Machuca, J. J, Morente, S. 2017, Highlights of Spanish Astrophysics IX, Proceedings of the XII Scientific Meeting of the Spanish Astronomical Society, 715

[6] Caballero, J. A., Arias, A., Benito, G. et al. 2020, Contributions to the XIV.0 Scientific Meeting (virtual) of the Spanish Astronomical Society, 247